

Douglas C. Montgomery • George C. Runger

# APPLIED STATISTICS AND PROBABILITY FOR ENGINEERS

7th Edition

WILEY

A microscopic view of a DNA microarray, showing numerous small, colorful spots arranged in a grid pattern. The spots are illuminated, creating a bright, glowing effect in the center of the array.



# Applied Statistics and Probability for Engineers

**Seventh Edition**

**DOUGLAS C. MONTGOMERY**

Arizona State University

**GEORGE C. RUNGER**

Arizona State University

**WILEY**

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Rebecca, Elisa, George and Taylor

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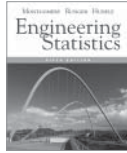
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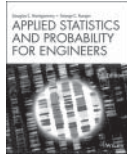
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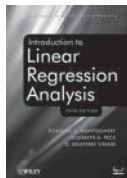
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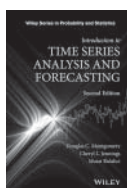


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# Preface

## INTENDED AUDIENCE

This is an introductory textbook for a first course in applied statistics and probability for undergraduate students in engineering and the physical or chemical sciences. These individuals play a significant role in designing and developing new products and manufacturing systems and processes, and they also improve existing systems. Statistical methods are an important tool in these activities because they provide the engineer with both descriptive and analytical methods for dealing with the variability in observed data. Although many of the methods we present are fundamental to statistical analysis in other disciplines, such as business and management, the life sciences, and the social sciences, we have elected to focus on an engineering-oriented audience. We believe that this approach will best serve students in engineering and the chemical/physical sciences and will allow them to concentrate on the many applications of statistics in these disciplines. We have worked hard to ensure that our examples and exercises are engineering- and science-based, and in almost all cases we have used examples of real data—either taken from a published source or based on our consulting experiences.

We believe that engineers in all disciplines should take at least one course in statistics. Unfortunately, because of other requirements, most engineers will only take one statistics course. This book can be used for a single course, although we have provided enough material for two courses in the hope that more students will see the important applications of statistics in their everyday work and elect a second course. We believe that this book will also serve as a useful reference.

We have retained the relatively modest mathematical level of the first five editions. We have found that engineering students who have completed one or two semesters of calculus and have some knowledge of matrix algebra should have no difficulty reading all of the text. It is our intent to give the reader an understanding of the methodology and how to apply it, not the mathematical theory. We have made many enhancements in this edition, including reorganizing and rewriting major portions of the book and adding a number of new exercises.

## ORGANIZATION OF THE BOOK

Perhaps the most common criticism of engineering statistics texts is that they are too long. Both instructors and students complain that it is impossible to cover all of the topics in the book in one or even two terms. For authors, this is a serious issue because there is great variety in both the content and level of these courses, and the decisions about what material to delete without limiting the value of the text are not easy. Decisions about which topics to include in this edition were made based on a survey of instructors.

Chapter 1 is an introduction to the field of statistics and how engineers use statistical methodology as part of the engineering problem-solving process. This chapter also introduces the reader to some engineering applications of statistics, including building empirical models, designing engineering experiments, and monitoring manufacturing processes. These topics are discussed in more depth in subsequent chapters.

Chapters 2, 3, 4, and 5 cover the basic concepts of probability, discrete and continuous random variables, probability distributions, expected values, joint probability distributions, and independence. We have given a reasonably complete treatment of these topics but have avoided many of the mathematical or more theoretical details.

Chapter 6 begins the treatment of statistical methods with random sampling; data summary and description techniques, including stem-and-leaf plots, histograms, box plots, and probability plotting; and several types of time series plots. Chapter 7 discusses sampling distributions, the

central limit theorem, and point estimation of parameters. This chapter also introduces some of the important properties of estimators, the method of maximum likelihood, the method of moments, and Bayesian estimation.

Chapter 8 discusses interval estimation for a single sample. Topics included are confidence intervals for means, variances or standard deviations, proportions, prediction intervals, and tolerance intervals. Chapter 9 discusses hypothesis tests for a single sample. Chapter 10 presents tests and confidence intervals for two samples. This material has been extensively rewritten and reorganized. There is detailed information and examples of methods for determining appropriate sample sizes. We want the student to become familiar with how these techniques are used to solve real-world engineering problems and to get some understanding of the concepts behind them. We give a logical, heuristic development of the procedures rather than a formal, mathematical one. We have also included some material on nonparametric methods in these chapters.

Chapters 11 and 12 present simple and multiple linear regression including model adequacy checking and regression model diagnostics and an introduction to logistic regression. We use matrix algebra throughout the multiple regression material (Chapter 12) because it is the only easy way to understand the concepts presented. Scalar arithmetic presentations of multiple regression are awkward at best, and we have found that undergraduate engineers are exposed to enough matrix algebra to understand the presentation of this material.

Chapters 13 and 14 deal with single- and multifactor experiments, respectively. The notions of randomization, blocking, factorial designs, interactions, graphical data analysis, and fractional factorials are emphasized. Chapter 15 introduces statistical quality control, emphasizing the control chart and the fundamentals of statistical process control.

## WHAT'S NEW IN THIS EDITION

It is highly gratifying that the sixth edition of the text was the most widely used edition in its history. For this seventh edition, we focused on revisions and enhancements aimed at *lowering costs to the students, better engaging them in the learning process, and providing increased support for instructors.*

### Lower-Cost Product Formats

According to the publishing industry's most trusted market research firm, Outsell, 2015 represented a turning point in text usage: for the first time, student preference for digital learning materials was higher than for print, and the increase in preference for digital has been accelerating since.

While print remains important for many students as a pedagogical tool, the seventh edition is being delivered in forms that emphasize support for learning from digital materials. *All forms we are providing dramatically reduce the cost to students compared to the sixth edition.* These forms are:

- **WileyPLUS stand-alone.** The WileyPLUS course incorporates the full text and delivers a wealth of integrated, interactive learning aids. See the detailed list of the learning aids below.
- **Stand-alone e-text now with significant enhancements.** The e-text format for the seventh edition adds approximately 100 practice problems with solutions, several hundred basic "Check Your Understanding" exercises for student self-quizzing, and all of the videos included in the WileyPLUS course.
- The two electronic formats listed above can be bundled with an **Abridged Print Companion** that includes a loose-leaf copy of the entire text minus the exercises for a nominal additional cost over the any of the value-priced forms listed above.

To explore these options further or to discuss other options, contact your Wiley account manager ([www.wiley.com/go/whosmyrep](http://www.wiley.com/go/whosmyrep)) or visit the product information page for this text on [wiley.com](http://wiley.com). Instructors may also order bound print companions for the bundled package by contacting their Wiley account representative.

## FEATURED IN THIS BOOK

### Definitions, Key Concepts, and Equations

Throughout the text, definitions and key concepts and equations are highlighted by a box to emphasize their importance.

### Learning Objectives

Learning Objectives at the start of each chapter guide the students in what they are expected to take away from this chapter and serve as a study reference.

#### LEARNING OBJECTIVES

After careful study of this chapter, you should be able to do the following:

1. Determine probabilities from probability density functions
2. Determine probabilities from cumulative distribution functions and cumulative distribution functions from probability density functions, and the reverse
3. Calculate means and variances for continuous random variables
4. Understand the assumptions for some common continuous probability distributions
5. Calculate probabilities, determine means and variances for some common continuous probability distributions
6. Approximate probabilities for binomial and Poisson distributions

### Seven-Step Procedure for Hypothesis Testing

The text introduces a sequence of seven steps in applying hypothesis-testing methodology and explicitly exhibits this procedure in examples.

#### 4.2 Cumulative Distribution Functions

An alternative method to describe the distribution of a discrete random variable can also be used for continuous random variables.

##### Cumulative Distribution Function

The **cumulative distribution function** of a continuous random variable  $X$  is

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du \quad (4.3)$$

for  $-\infty < x < \infty$ .

The cumulative distribution function is defined for all real numbers.

##### EXAMPLE 4.3 | Electric Current

For the copper current measurement in Example 4.1, the cumulative distribution function of the random variable  $X$  consists of three expressions. If  $x < 4.9$ ,  $f(x) = 0$ . Therefore,

$$F(x) = 0, \quad \text{for } x < 4.9$$

and

$$F(x) = \int_{-\infty}^x f(u) du = 5x - 24.5, \quad \text{for } 4.9 \leq x < 5.1$$

Finally,

$$F(x) = \int_{-\infty}^x f(u) du = 1, \quad \text{for } 5.1 \leq x$$

Therefore,

$$F(x) = \begin{cases} 0 & x < 4.9 \\ 5x - 24.5 & 4.9 \leq x < 5.1 \\ 1 & 5.1 \leq x \end{cases}$$

The plot of  $F(x)$  is shown in Figure 4.6.

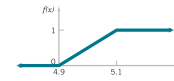


FIGURE 4.6  
Cumulative distribution function for Example 4.3.

Notice that in the definition of  $F(x)$ , any  $<$  can be changed to  $\leq$  and vice versa. That is, in Example 4.3  $F(x)$  can be defined as either  $5x - 24.5$  or 0 at the end-point  $x = 4.9$ , and  $F(x)$  can be defined as either  $5x - 24.5$  or 1 at the end-point  $x = 5.1$ . In other words,  $F(x)$  is a continuous function. For a discrete random variable,  $F(x)$  is not a continuous function. Sometimes a continuous random variable is defined as one that has a continuous cumulative distribution function.

The probability density function of a continuous random variable can be determined from the cumulative distribution function by differentiating. The fundamental theorem of calculus states that

$$\frac{d}{dx} \int_{-\infty}^x f(u) du = f(x)$$

##### Probability Density Function from the Cumulative Distribution Function

Given  $F(x)$ ,

$$f(x) = \frac{dF(x)}{dx}$$

as long as the derivative exists.

#### 9.1.6 General Procedure for Hypothesis Tests

This chapter develops hypothesis-testing procedures for many practical problems. Use of the following sequence of steps in applying hypothesis-testing methodology is recommended.

1. **Parameter of interest:** From the problem context, identify the parameter of interest.
2. **Null hypothesis,  $H_0$ :** State the null hypothesis,  $H_0$ .
3. **Alternative hypothesis,  $H_1$ :** Specify an appropriate alternative hypothesis,  $H_1$ .
4. **Test statistic:** Determine an appropriate test statistic.
5. **Reject  $H_0$  if:** State the rejection criteria for the null hypothesis.
6. **Computations:** Compute any necessary sample quantities, substitute these into the equation for the test statistic, and compute that value.
7. **Draw conclusions:** Decide whether or not  $H_0$  should be rejected and report that in the problem context.



### Figures

Numerous figures throughout the text illustrate statistical concepts in multiple formats.

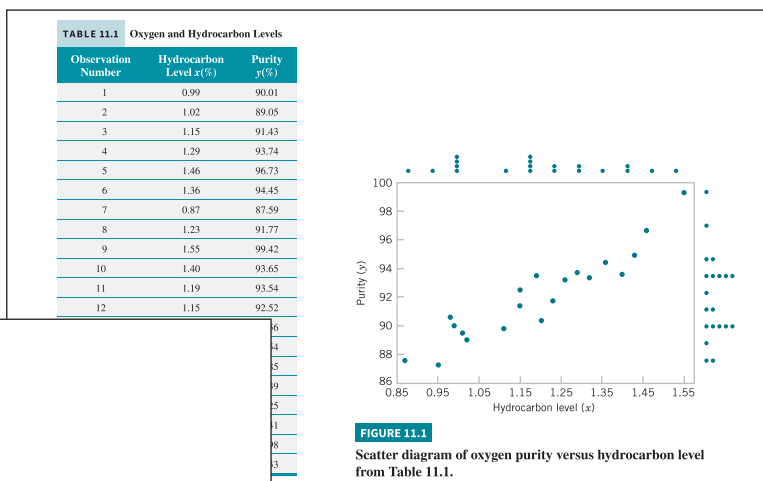


FIGURE 11.1 Scatter diagram of oxygen purity versus hydrocarbon level from Table 11.1.

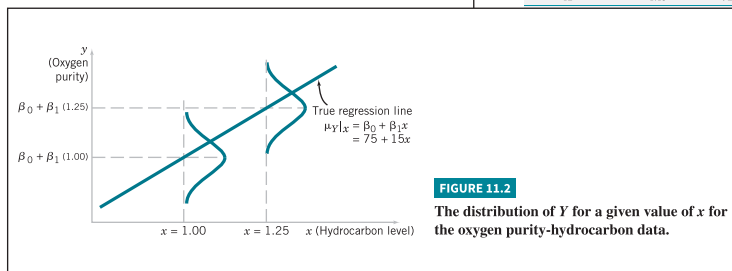


FIGURE 11.2 The distribution of  $Y$  for a given value of  $x$  for the oxygen purity-hydrocarbon data.

### Computer Output

Example throughout the book, use computer output to illustrate the role of modern statistical software.

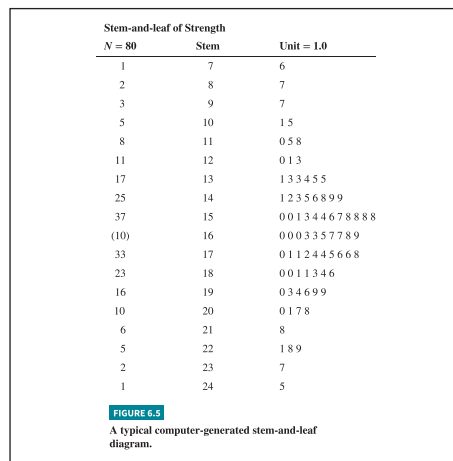


FIGURE 6.5 A typical computer-generated stem-and-leaf diagram.

### Example Problems

A set of example problems provides the student with detailed solutions and comments for interesting, real-world situations. Brief practical interpretations have been added in this edition.

**EXAMPLE 10.1 | Paint Drying Time**

A product developer is interested in reducing the drying time of a primer paint. Two formulations of the paint are tested; formulation 1 is the standard chemistry, and formulation 2 has a new drying ingredient that should reduce the drying time. From experience, it is known that the standard deviation of drying time is 8 minutes, and this inherent variability should be unaffected by the addition of the new ingredient. Ten specimens are painted with formulation 1, and another 10 specimens are painted with formulation 2; the 20 specimens are painted in random order. The two sample average drying times are  $\bar{x}_1 = 121$  minutes and  $\bar{x}_2 = 112$  minutes, respectively. What conclusions can the product developer draw about the effectiveness of the new ingredient, using  $\alpha = 0.05$ ?

We apply the seven-step procedure to this problem as follows:

- Parameter of interest:** The quantity of interest is the difference in mean drying times,  $\mu_1 - \mu_2$ , and  $\Delta_0 = 0$ .
- Null hypothesis:**  $H_0: \mu_1 - \mu_2 = 0$ , or  $H_0: \mu_1 = \mu_2$ .
- Alternative hypothesis:**  $H_1: \mu_1 > \mu_2$ . We want to reject  $H_0$  if the new ingredient reduces mean drying time.
- Test statistic:** The test statistic is
 
$$z_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$
 where  $\sigma_1^2 = \sigma_2^2 = (8)^2 = 64$  and  $n_1 = n_2 = 10$ .
- Reject  $H_0$  if:** Reject  $H_0: \mu_1 = \mu_2$  if the  $P$ -value is less than 0.05.
- Computations:** Because  $\bar{x}_1 = 121$  minutes and  $\bar{x}_2 = 112$  minutes, the test statistic is
 
$$z_0 = \frac{121 - 112}{\sqrt{\frac{(8)^2}{10} + \frac{(8)^2}{10}}} = 2.52$$
- Conclusion:** Because  $z_0 = 2.52$ , the  $P$ -value is  $P = 1 - \Phi(2.52) = 0.0059$ , so we reject  $H_0$  at the  $\alpha = 0.05$  level.

**Practical Interpretation:** We conclude that adding the new ingredient to the paint significantly reduces the drying time. This is a strong conclusion.

## Exercises

Each chapter has an extensive collection of exercises, including **end-of-section exercises** that emphasize the material in that section, **supplemental exercises** at the end of the chapter that cover the scope of chapter topics and require the student to make a decision about the approach they will use to solve the problem, and **mind-expanding exercises** that often require the student to extend the text material somewhat or to apply it in a novel situation. Answers are provided to most odd-numbered exercises in Appendix C in the text, and the *WileyPLUS* online learning environment includes for students complete detailed solutions to selected exercises.

## Important Terms and Concepts

At the end of each chapter is a list of important terms and concepts for an easy self-check and study tool.

### Exercises for Section 2.2

- WP** Problem available in WileyPLUS at instructor's discretion.  
**WP GO Tutorial** Tutoring problem available in WileyPLUS at instructor's discretion.  
**SS** Student solution available in WileyPLUS.  
**VS** Student video solution available in WileyPLUS.

2.2.1 **WP** without space for board.

### Supplemental Exercises for Chapter 2

- WP** Problem available in WileyPLUS at instructor's discretion.  
**WP GO Tutorial** Tutoring problem available in WileyPLUS at instructor's discretion.  
**SS** Student solution available in WileyPLUS.  
**VS** Student video solution available in WileyPLUS.

**2.S4 WP** Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2% and 1%, respectively, of the sample shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment?

**2.S5 WP SS** Samples of a cast aluminum part are classified on the basis of surface finish (in microinches) and edge finish. The results of 100 parts are summarized as follows: selected randomly, without replacement, from the lot of 40. Let  $A$  be the event that the first casting selected is from the local supplier, and let  $B$  denote the event that the second casting is selected from the local supplier. Determine:

- a.  $P(A)$    b.  $P(B|A)$    c.  $P(A \cap B)$    d.  $P(A \cup B)$

Suppose that 3 castings are selected at random, without replacement, from the lot of 40. In addition to the definitions of events  $A$  and  $B$ , let  $C$  denote the event that the third casting selected is from the local supplier. Determine:

- e.  $P(A \cap B \cap C)$    f.  $P(A \cap B \cap C')$

**2.S7** If  $A$ ,  $B$ , and  $C$  are mutually exclusive events, is it possible for  $P(A) = 0.3$ ,  $P(B) = 0.4$ , and  $P(C) = 0.5$ ? Why or why not?

**2.S8 WP** Incoming calls to a customer service center are classified as complaints (75% of calls) or requests for information (25% of calls). Of the complaints, 40% deal with computer equipment that does not respond and 57% deal with incomplete software installation; in the remaining 3% of complaints, the user has improperly followed the installation instructions. The requests for

**2.2.9** In the laboratory analysis of samples from a chemical process, five samples from the process are analyzed daily. In addition, a control sample is analyzed twice each day to check the calibration of the laboratory instruments.

- a. How many different sequences of process and control

Surface finish	Edge Finish	
	Excellent	Good
Excellent	80	2
Good	10	8

Let  $A$  denote the event that a sample has excellent surface finish, and let  $B$  denote the event that a sample has excellent edge finish. If a part is selected at random, determine the following probabilities:

- a.  $P(A)$    b.  $P(B)$    c.  $P(A')$   
 d.  $P(A \cap B)$    e.  $P(A \cup B)$    f.  $P(A' \cup B)$

**2.S6 WP** A lot contains 15 castings from a local supplier and 25 castings from a supplier in the next state. Two castings are

Use a tree diagram to determine the following:

- a.  $P(B)$    b.  $P(A|B)$    c.  $P(A|B')$   
 d.  $P(A \cap B)$    e.  $P(A \cap B')$    f.  $P(A)$

**g.** What is the probability that the useful life of a laser exceeds five years?

**h.** What is the probability that a laser that failed before five years came from a product used for backup?

**2.S11 WP SS** A congested computer network has a 0.002 probability of losing a data packet, and packet losses are independent events. A lost packet must be resent.

- a. What is the probability that an e-mail message with 100 packets will need to be resent?  
 b. What is the probability that an e-mail message with 3 packets will need exactly 1 to be resent?  
 c. If 10 e-mail messages are sent, each with 100 packets, what is the probability that at least 1 message will need some packets to be resent?

**2.S12 WP GO Tutorial** An electronic storage device uses an error recovery procedure that requires an immediate satisfactory read-back of any written data. If the read-back is not successful after

## Important Terms and Concepts

Bayes estimator	Moment estimator	Posterior distribution
Bias in parameter estimation	Normal distribution as the sampling distribution of the difference in two sample means	Prior distribution
Bootstrap method	Normal distribution as the sampling distribution of a sample mean	Sample moments
Central limit theorem	Parameter estimation	Sampling distribution
Estimator versus estimate	Point estimator	Standard error and estimated standard error of an estimator
Likelihood function	Population or distribution moments	Statistic
Maximum likelihood estimator		Statistical inference
Mean squared error of an estimator		Unbiased estimator
Minimum variance unbiased estimator		

## Meticulous Ensuring of Accuracy

There is always room for improvement in this area for richly mathematical texts. We appreciate the input of sixth edition adopters and made achieving the greatest possible accuracy of every element of the seventh edition a very high-priority objective. We have been rigorous in reviewing the retained and new content in the text and reviewing and testing the gradable exercises in the WileyPLUS course. We thank the colleagues whom Wiley engaged to execute these activities: (1) the entire text, including all exposition, examples, exercises, and answers have been read and checked for accuracy, including calculations; (2) answers and solutions to all exercises have been checked via independent solution of problems; (3) all questions and exercises in the WileyPLUS course have been checked with a focus on verifying the answers, correcting any conflicting input instructions, and improving the precision of the numerical answers.

## Increased Support for Student Engagement

The rapidly increasing use of approaches like flipped classrooms and online course delivery has motivated us to enhance the features for student self-study, especially those that can better engage

students. The following new and expanded interactive features are available in both the WileyPLUS course and the interactive e-text:

- *Video-based examples and problem walk-throughs.* There are now over 100 video solutions and about 40 example videos.
- *Check Your Understanding* exercises are placed at the end of each appropriate section in the WileyPLUS e-text. These simple practice exercises allow students to test whether they comprehend the material they have just read. They can also be used, if the instructor prefers, to assess student readiness to apply concepts and techniques. The program contains over 600 Check Your Understanding questions.

## Greater Selection of Assignable Exercises

The seventh edition contains over 1800 exercises, an increase of about 25%. Over 1100 of these exercises are available to the students and appear in the instructor desk copy, e-text version of the text, and about 750 of these exercises have been reserved for instructors in WileyPlus. The e-text and WileyPLUS versions have also added about 300 new exercises to the Check Your Understanding program that consists of about 600 questions. These questions are integrated into the appropriate point in the text exposition to provide students with the ability to test their comprehension of the material just presented before proceeding. In the WileyPLUS course the Check Your Understanding exercises are assignable, enabling instructors to assess student readiness.

## Instructor-Only and Algorithmic Exercises for More Exercise Security

The wide and inexpensive availability of solutions to all published textbook problems has added a huge challenge for instructors looking to assess learning. Wiley is embarking on a comprehensive strategy to provide original and reconfigured problems that can decrease the proliferation of problem solutions on the Web. As a first step in this direction, the instructor evaluation/desk copy of this text includes about 750 problems that are *reserved for the instructor only—they do not appear in the student version*. These problems are provided in the instructor desk copy for easy reference. All of these reserved questions appear as assignable problems in WileyPLUS. About 40% of these problems are entirely brand new. The other 60% of these problems have been revised from earlier editions and are now only available as assignable problems. The WileyPLUS course also features *algorithmic homework problems*, machine-gradable problems in which each student has unique values for key variables.

## Exercises Collected in the Back of the Book in Instructor Desk Copy

Many instructors have requested that we curtail the practice of frequent revisions of the text. At the same time, the demand for fresh and new problems among instructors is quite high. The exercises in the seventh edition are collected in the back of the instructor desk copy as well as the student e-text and WileyPLUS e-text. We have done this in effort to be able to generate entirely new problem collections that can be adopted without altering the contents of the book. We expect that revisions going forward will be confined largely to expansion of learning aids in the digital forms of the text and updates of the problems and to elongate the life of the print text.

## STUDENT RESOURCES

- **Data Sets** Data sets for all examples and exercises in the text. Visit the student section of the book Web site at [www.wiley.com/college/montgomery](http://www.wiley.com/college/montgomery) to access these materials.

## INSTRUCTOR RESOURCES

The following resources are available only to instructors who adopt the text:

- **Solutions Manual** All solutions to the exercises in the text.
- **Data Sets** Data sets for all examples and exercises in the text.
- **Image Gallery of Text Figures**
- **Section on Logistic Regression**

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## COMPUTER SOFTWARE

We have used several different packages, including Excel, to demonstrate computer usage. Minitab can be used for most exercises. A student version of Minitab is available as an option to purchase in a set with this text. Student versions of software often do not have all the functionality that full versions do. Consequently, student versions may not support all the concepts presented in this text. If you would like to adopt for your course the set of this text with the student version of Minitab, please contact your local Wiley representative at [www.wiley.com/college/rep](http://www.wiley.com/college/rep).

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## COURSE SYLLABUS SUGGESTIONS

This is a very flexible textbook because instructors' ideas about what should be in a first course on statistics for engineers vary widely, as do the abilities of different groups of students. Therefore, we hesitate to give too much advice, but will explain how we use the book.

We believe that a first course in statistics for engineers should be primarily an applied statistics course, not a probability course. In our one-semester course we cover all of Chapter 1 (in one or two lectures); overview the material on probability, putting most of the emphasis on the normal distribution (six to eight lectures); discuss most of Chapters 6 through 10 on confidence intervals and tests (twelve to fourteen lectures); introduce regression models in Chapter 11 (four lectures); give an introduction to the design of experiments from Chapters 13 and 14 (six lectures); and present the basic concepts of statistical process control, including the Shewhart control chart from Chapter 15 (four lectures). This leaves about three to four periods for exams and review. Let us emphasize that the purpose of this course is to introduce engineers to how statistics can be used to solve real-world engineering problems, not to weed out the less mathematically gifted students. This course is not the “baby math-stat” course that is all too often given to engineers.

If a second semester is available, it is possible to cover the entire book, including much of the supplemental material, if appropriate for the audience. It would also be possible to assign and work many of the homework problems in class to reinforce the understanding of the concepts. Obviously, multiple regression and more design of experiments would be major topics in a second course.

## USING THE COMPUTER

In practice, engineers use computers to apply statistical methods to solve problems. Therefore, we strongly recommend that the computer be integrated into the class. Throughout the book we have presented typical example of the output that can be obtained with modern statistical software. In teaching, we have used a variety of software packages, including Minitab, Statgraphics, JMP, and Statistica. We did not clutter up the book with operational details of these different packages because how the instructor integrates the software into the class is ultimately more important than which package is used. All text data are available in electronic form on the textbook Web site. We have marked these problems with a special icon in the margin.

In our own classrooms, we use the computer in almost every lecture and demonstrate how the technique is implemented in software as soon as it is discussed in the lecture. Many educational institutions have file licenses for statistics software and students can access it for class use. Student versions of many statistical software packages are available at low cost, and students can either purchase their own copy or use the products available through the institution. We have found that greatly improves the pace of the course and student understanding of the material.

Users should be aware that final answers may differ slightly due to different numerical precision and rounding protocols among softwares.

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**DOUGLAS C. MONTGOMERY**

**GEORGE C. RUNGER**

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# The Role of Statistics in Engineering

## CHAPTER 1



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### LEARNING OBJECTIVES

After careful study of this chapter, you should be able to do the following:

1. Identify the role that statistics can play in the engineering problem-solving process
2. Discuss how variability affects the data collected and used for making engineering decisions
3. Explain the difference between enumerative and analytical studies
4. Discuss the different methods that engineers use to collect data
5. Identify the advantages that designed experiments have in comparison to other methods of collecting engineering data
6. Explain the differences between mechanistic models and empirical models
7. Discuss how probability and probability models are used in engineering and science

### CHAPTER OUTLINE

#### 1.1 The Engineering Method and Statistical Thinking

##### 1.1.1 Variability

##### 1.1.2 Populations and Samples

#### 1.2 Collecting Engineering Data

##### 1.2.1 Basic Principles

##### 1.2.2 Retrospective Study

##### 1.2.3 Observational Study

##### 1.2.4 Designed Experiments

##### 1.2.5 Observing Processes Over Time

#### 1.3 Mechanistic and Empirical Models

#### 1.4 Probability and Probability Models

**Statistics** is a science that helps us make decisions and draw conclusions in the presence of variability. For example, civil engineers working in the transportation field are concerned about the capacity of regional highway systems. A typical problem related to transportation would involve data regarding this specific system's number of nonwork, home-based trips, the number of persons per household, and the number of vehicles per household. The objective would be to produce a trip-generation model relating trips to the number of persons per household and the number of vehicles per household. A statistical technique called *regression analysis* can be used to construct this model. The trip-generation model is an important tool for transportation systems planning. Regression methods are among the most widely used statistical techniques in engineering. They are presented in Chapters 11 and 12.

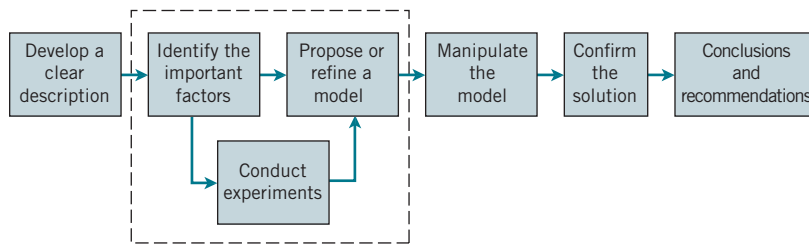
The hospital emergency department (ED) is an important part of the healthcare delivery system. The process by which patients arrive at the ED is highly variable and can depend on the hour of the day and the day of the week, as well as on longer-term cyclical variations. The service process is also highly variable, depending on the types of services that the patients require, the number of patients in the ED, and how the ED is staffed and organized. An ED's capacity is also limited; consequently, some patients experience long waiting times. How long do patients wait, on average? This is an important question for healthcare providers. If waiting times become excessive, some patients will leave without receiving treatment (LWOT). Patients who LWOT do not have their medical concerns addressed and are at risk for further problems and complications. Therefore, another important question is: What proportion of patients LWOT from the ED? These questions can be solved by employing probability models to describe the ED, and from these models very precise estimates of waiting times and the number of patients who LWOT can be obtained. Probability models that can be used to solve these types of problems are discussed in Chapters 2 through 5.

The concepts of probability and statistics are powerful ones and contribute extensively to the solutions of many types of engineering problems. You encounter many examples of these applications in this book.

## 1.1 The Engineering Method and Statistical Thinking

An engineer is someone who solves problems of interest to society by the efficient application of scientific principles. Engineers accomplish this by either refining an existing product or process or by designing a new product or process that meets customers' needs. The **engineering, or scientific, method** is the approach to formulating and solving these problems. The steps in the engineering method are as follows:

1. Develop a clear and concise description of the problem.
2. Identify, at least tentatively, the important factors that affect this problem or that may play a role in its solution.
3. Propose a model for the problem, using scientific or engineering knowledge of the phenomenon being studied. State any limitations or assumptions of the model.
4. Conduct appropriate experiments and collect data to test or validate the tentative model or conclusions made in steps 2 and 3.
5. Refine the model on the basis of the observed data.
6. Manipulate the model to assist in developing a solution to the problem.



**FIGURE 1.1** The engineering method.

7. Conduct an appropriate experiment to confirm that the proposed solution to the problem is both effective and efficient.
8. Draw conclusions or make recommendations based on the problem solution.

The steps in the engineering method are shown in Figure 1.1. Many engineering sciences employ the engineering method: the mechanical sciences (statics, dynamics), fluid science, thermal science, electrical science, and the science of materials. Notice that the engineering method features a strong interplay among the problem, the factors that may influence its solution, a model of the phenomenon, and experimentation to verify the adequacy of the model and the proposed solution to the problem. Steps 2–4 in Figure 1.1 are enclosed in a box, indicating that several cycles or iterations of these steps may be required to obtain the final solution. Consequently, engineers must know how to efficiently plan experiments, collect data, analyze and interpret the data, and understand how the observed data relate to the model they have proposed for the problem under study.

The field of **statistics** deals with the collection, presentation, analysis, and use of data to make decisions, solve problems, and design products and processes. In simple terms, **statistics is the science of data**. Because many aspects of engineering practice involve working with data, obviously knowledge of statistics is just as important to an engineer as are the other engineering sciences. Specifically, statistical techniques can be powerful aids in designing new products and systems, improving existing designs, and designing, developing, and improving production processes.

### 1.1.1 Variability

Statistical methods are used to help us describe and understand **variability**. By variability, we mean that successive observations of a system or phenomenon do not produce exactly the same result. We all encounter variability in our everyday lives, and **statistical thinking** can give us a useful way to incorporate this variability into our decision-making processes. For example, consider the gasoline mileage performance of your car. Do you always get exactly the same mileage performance on every tank of fuel? Of course not—in fact, sometimes the mileage performance varies considerably. This observed variability in gasoline mileage depends on many factors, such as the type of driving that has occurred most recently (city versus highway), the changes in the vehicle’s condition over time (which could include factors such as tire inflation, engine compression, or valve wear), the brand and/or octane number of the gasoline used, or possibly even the weather conditions that have been recently experienced. These factors represent potential **sources of variability** in the system. Statistics provides a framework for describing this variability and for learning about which potential sources of variability are the most important or which have the greatest impact on the gasoline mileage performance.

The following example illustrates how we encounter variability in dealing with engineering problems.

### EXAMPLE 1.1 | Pull-off Force Variability

Suppose that an engineer is designing a nylon connector to be used in an automotive engine application. The engineer is considering establishing the design specification on wall thickness at  $3/32$  inch but is somewhat uncertain about the effect of this decision on the connector pull-off force. If the pull-off force

is too low, the connector may fail when it is installed in an engine. Eight prototype units are produced and their pull-off forces measured, resulting in the following data (in pounds): 12.6, 12.9, 13.4, 12.3, 13.6, 13.5, 12.6, 13.1. As we anticipated, not all of the prototypes have the same pull-off force. We say that there is variability in the pull-off force measurements.

Because the pull-off force measurements exhibit variability, we consider the pull-off force to be a **random variable**. A convenient way to think of a random variable, say  $X$ , that represents a measurement is by using the model

$$X = \mu + \epsilon \quad (1.1)$$

where  $\mu$  is a constant and  $\epsilon$  is a random disturbance. The constant remains the same with every measurement, but small changes in the environment, variance in test equipment, differences in the individual parts themselves, and so forth change the value of  $\epsilon$ . If there were no disturbances,  $\epsilon$  would always equal zero and  $X$  would always be equal to the constant  $\mu$ . However, this never happens in the real world, so the actual measurements  $X$  exhibit variability. We often need to describe, quantify, and ultimately reduce variability.

Figure 1.2 presents a **dot diagram** of these data. The dot diagram is a very useful plot for displaying a small body of data—say, up to about 20 observations. This plot allows us to easily see two features of the data: the **location**, or the middle, and the **scatter** or **variability**. When the number of observations is small, it is usually difficult to identify any specific patterns in the variability, although the dot diagram is a convenient way to see any unusual data features.

The need for statistical thinking arises often in the solution of engineering problems. Consider the engineer designing the connector. From testing the prototypes, he knows that the average pull-off force is 13.0 pounds. However, he thinks that this may be too low for the intended application, so he decides to consider an alternative design with a thicker wall,  $1/8$  inch in thickness. Eight prototypes of this design are built, and the observed pull-off force measurements are 12.9, 13.7, 12.8, 13.9, 14.2, 13.2, 13.5, and 13.1. The average is 13.4. Results for both samples are plotted as dot diagrams in Figure 1.3. This display gives the impression that increasing the wall thickness has led to an increase in pull-off force. However, there are some obvious questions to ask. For instance, how do we know that another sample of prototypes will not give different results? Is a sample of eight prototypes adequate to give reliable results? If we use the test results obtained so far to conclude that increasing the wall thickness increases the strength, what risks are associated with this decision? For example, is it possible that the apparent increase in pull-off force observed in the thicker prototypes is due only to the inherent variability in the system and that increasing the thickness of the part (and its cost) really has no effect on the pull-off force?



FIGURE 1.2

Dot diagram of the pull-off force data when wall thickness is  $3/32$  inch.

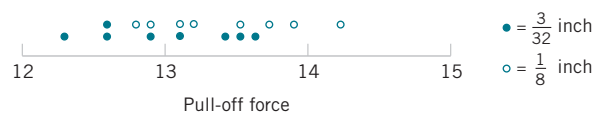


FIGURE 1.3

Dot diagram of pull-off force for two wall thicknesses.

### 1.1.2 Populations and Samples

Often, physical laws (such as Ohm's law and the ideal gas law) are applied to help design products and processes. We are familiar with this reasoning from general laws to specific cases. But it is also important to reason from a specific set of measurements to more general cases to answer the previous questions. This reasoning comes from a **sample** (such as the eight connectors) to a **population** (such as the connectors that will be in the products that are sold to customers). The reasoning is referred to as **statistical inference**. See Figure 1.4. Historically, measurements were obtained from a sample of people and generalized to a population, and the terminology has remained. Clearly, reasoning based on measurements from some objects to measurements on all objects can result in errors (called *sampling errors*). However, if the sample is selected properly, these risks can be quantified and an appropriate sample size can be determined.

## 1.2 Collecting Engineering Data

### 1.2.1 Basic Principles

In the previous subsection, we illustrated some simple methods for summarizing data. Sometimes the data are all of the observations in the population. This results in a **census**. However, in the engineering environment, the data are almost always a **sample** that has been selected from the population. Three basic methods of collecting data are

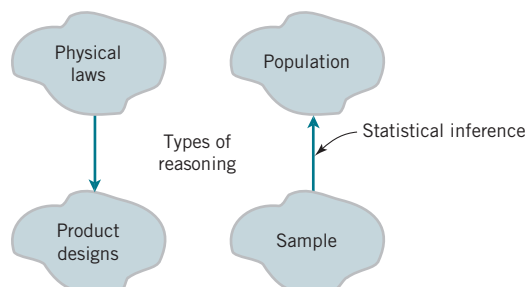
- A **retrospective study** using historical data
- An **observational study**
- A **designed experiment**

An effective data-collection procedure can greatly simplify the analysis and lead to improved understanding of the population or process that is being studied. We now consider some examples of these data-collection methods.

### 1.2.2 Retrospective Study

Montgomery, Peck, and Vining (2012) describe an acetone-butyl alcohol distillation column for which concentration of acetone in the distillate (the output product stream) is an important variable. Factors that may affect the distillate are the reboil temperature, the condensate temperature, and the reflux rate. Production personnel obtain and archive the following records:

- The concentration of acetone in an hourly test sample of output product
- The reboil temperature log, which is a record of the reboil temperature over time
- The condenser temperature controller log
- The nominal reflux rate each hour



**FIGURE 1.4**

**Statistical inference is one type of reasoning.**

The reflux rate should be held constant for this process. Consequently, production personnel change this very infrequently.

A retrospective study would use either all or a sample of the historical process data archived over some period of time. The study objective might be to discover the relationships among the two temperatures and the reflux rate on the acetone concentration in the output product stream. However, this type of study presents some problems:

1. We may not be able to see the relationship between the reflux rate and acetone concentration because the reflux rate did not change much over the historical period.
2. The archived data on the two temperatures (which are recorded almost continuously) do not correspond perfectly to the acetone concentration measurements (which are made hourly). It may not be obvious how to construct an approximate correspondence.
3. Production maintains the two temperatures as closely as possible to desired targets or set points. Because the temperatures change so little, it may be difficult to assess their real impact on acetone concentration.
4. In the narrow ranges within which they do vary, the condensate temperature tends to increase with the reboil temperature. Consequently, the effects of these two process variables on acetone concentration may be difficult to separate.

As you can see, a retrospective study may involve a significant amount of **data**, but those data may contain relatively little useful **information** about the problem. Furthermore, some of the relevant data may be missing, there may be transcription or recording errors resulting in **outliers** (or unusual values), or data on other important factors may not have been collected and archived. In the distillation column, for example, the specific concentrations of butyl alcohol and acetone in the input feed stream are very important factors, but they are not archived because the concentrations are too hard to obtain on a routine basis. As a result of these types of issues, statistical analysis of historical data sometimes identifies interesting phenomena, but solid and reliable explanations of these phenomena are often difficult to obtain.

### 1.2.3 Observational Study

In an observational study, the engineer observes the process or population, disturbing it as little as possible, and records the quantities of interest. Because these studies are usually conducted for a relatively short time period, sometimes variables that are not routinely measured can be included. In the distillation column, the engineer would design a form to record the two temperatures and the reflux rate when acetone concentration measurements are made. It may even be possible to measure the input feed stream concentrations so that the impact of this factor could be studied.

Generally, an observational study tends to solve problems 1 and 2 and goes a long way toward obtaining accurate and reliable data. However, observational studies may not help resolve problems 3 and 4.

### 1.2.4 Designed Experiments

In a designed experiment, the engineer makes *deliberate* or *purposeful changes* in the controllable variables of the system or process, observes the resulting system output data, and then makes an inference or decision about which variables are responsible for the observed changes in output performance. The nylon connector example in Section 1.1 illustrates a designed experiment; that is, a deliberate change was made in the connector's wall thickness with the objective of discovering whether or not a stronger pull-off force could be obtained. Experiments designed with basic principles such as **randomization** are needed to establish **cause-and-effect** relationships.

Much of what we know in the engineering and physical-chemical sciences is developed through testing or experimentation. Often engineers work in problem areas in which no scientific



or engineering theory is directly or completely applicable, so experimentation and observation of the resulting data constitute the only way that the problem can be solved. Even when there is a good underlying scientific theory that we may rely on to explain the phenomena of interest, it is almost always necessary to conduct tests or experiments to confirm that the theory is indeed operative in the situation or environment in which it is being applied. Statistical thinking and statistical methods play an important role in planning, conducting, and analyzing the data from engineering experiments. Designed experiments play a very important role in engineering design and development and in the improvement of manufacturing processes.

For example, consider the problem involving the choice of wall thickness for the nylon connector. This is a simple illustration of a designed experiment. The engineer chose two wall thicknesses for the connector and performed a series of tests to obtain pull-off force measurements at each wall thickness. In this simple **comparative experiment**, the engineer is interested in determining whether there is any difference between the 3/32- and 1/8-inch designs. An approach that could be used in analyzing the data from this experiment is to compare the mean pull-off force for the 3/32-inch design to the mean pull-off force for the 1/8-inch design using statistical **hypothesis testing**, which is discussed in detail in Chapters 9 and 10. Generally, a **hypothesis** is a statement about some aspect of the system in which we are interested. For example, the engineer might want to know if the mean pull-off force of a 3/32-inch design exceeds the typical maximum load expected to be encountered in this application, say, 12.75 pounds. Thus, we would be interested in testing the hypothesis that the mean strength exceeds 12.75 pounds. This is called a **single-sample hypothesis-testing problem**. Chapter 9 presents techniques for this type of problem. Alternatively, the engineer might be interested in testing the hypothesis that increasing the wall thickness from 3/32 to 1/8 inch results in an increase in mean pull-off force. It is an example of a **two-sample hypothesis-testing problem**. Two-sample hypothesis-testing problems are discussed in Chapter 10.

Designed experiments offer a very powerful approach to studying complex systems, such as the distillation column. This process has three factors—the two temperatures and the reflux rate—and we want to investigate the effect of these three factors on output acetone concentration. A good experimental design for this problem must ensure that we can separate the effects of all three factors on the acetone concentration. The specified values of the three factors used in the experiment are called **factor levels**. Typically, we use a small number of levels such as two or three for each factor. For the distillation column problem, suppose that we use two levels, “high” and “low” (denoted +1 and −1, respectively), for each of the three factors. A very reasonable experiment design strategy uses every possible combination of the factor levels to form a basic experiment with eight different settings for the process. This type of experiment is called a **factorial experiment**. See Table 1.1 for this experimental design.

**TABLE 1.1** The Designed Experiment (Factorial Design) for the Distillation Column

Reboil Temp.	Condensate Temp.	Reflux Rate
−1	−1	−1
+1	−1	−1
−1	+1	−1
+1	+1	−1
−1	−1	+1
+1	−1	+1
−1	+1	+1
+1	+1	+1

FIGURE 1.5

The factorial design for the distillation column.

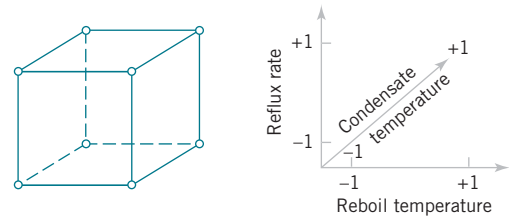


Figure 1.5 illustrates that this design forms a cube in terms of these high and low levels. With each setting of the process conditions, we allow the column to reach equilibrium, take a sample of the product stream, and determine the acetone concentration. We then can draw specific inferences about the effect of these factors. Such an approach allows us to proactively study a population or process.

An important advantage of factorial experiments is that they allow one to detect an **interaction** between factors. Consider only the two temperature factors in the distillation experiment. Suppose that the response concentration is poor when the reboil temperature is *low*, regardless of the condensate temperature. That is, the condensate temperature has no effect when the reboil temperature is *low*. However, when the reboil temperature is *high*, a *high* condensate temperature generates a good response, but a *low* condensate temperature generates a poor response. That is, the condensate temperature changes the response when the reboil temperature is *high*. The effect of condensate temperature depends on the setting of the reboil temperature, and these two factors are said to interact in this case. If the four combinations of *high* and *low* reboil and condensate temperatures were not tested, such an interaction would not be detected.

We can easily extend the factorial strategy to more factors. Suppose that the engineer wants to consider a fourth factor, type of distillation column. There are two types: the standard one and a newer design. Figure 1.6 illustrates how all four factors—reboil temperature, condensate temperature, reflux rate, and column design—could be investigated in a factorial design. Because all four factors are still at two levels, the experimental design can still be represented geometrically as a cube (actually, it's a *hypercube*). Notice that as in any factorial design, all possible combinations of the four factors are tested. The experiment requires 16 trials.

Generally, if there are  $k$  factors and each has two levels, a factorial experimental design will require  $2^k$  runs. For example, with  $k = 4$ , the  $2^4$  design in Figure 1.6 requires 16 tests. Clearly, as the number of factors increases, the number of trials required in a factorial experiment increases rapidly; for instance, eight factors each at two levels would require 256 trials. This quickly becomes unfeasible from the viewpoint of time and other resources. Fortunately, with four to five or more factors, it is usually unnecessary to test all possible combinations of factor levels. A **fractional factorial experiment** is a variation of the basic factorial arrangement in which only a subset of the factor combinations is actually tested. Figure 1.7 shows a fractional factorial experimental design for the distillation column. The circled test combinations in this figure are the only test combinations that need to be run. This experimental design

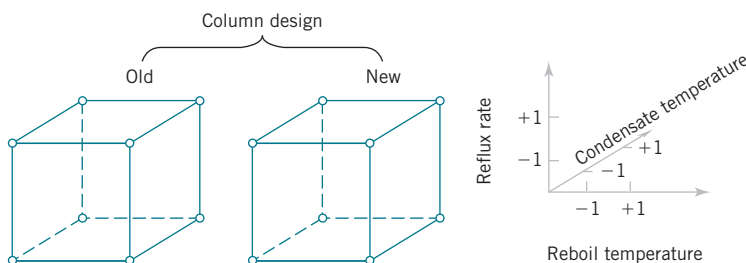
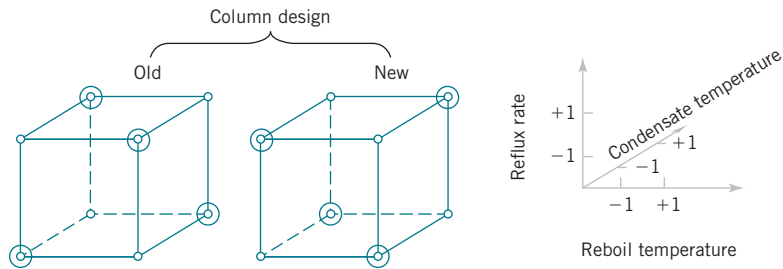


FIGURE 1.6 A four-factorial experiment for the distillation column.



**FIGURE 1.7** A fractional factorial experiment for the distillation column.

requires only 8 runs instead of the original 16; consequently it would be called a **one-half fraction**. This is an excellent experimental design in which to study all four factors. It will provide good information about the individual effects of the four factors and some information about how these factors interact.

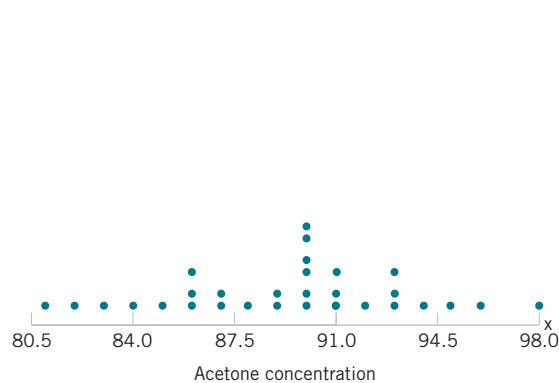
Factorial and fractional factorial experiments are used extensively by engineers and scientists in industrial research and development, where new technology, products, and processes are designed and developed and where existing products and processes are improved. Since so much engineering work involves testing and experimentation, it is essential that all engineers understand the basic principles of planning efficient and effective experiments. We discuss these principles in Chapter 13. Chapter 14 concentrates on the factorial and fractional factorials that we have introduced here.

### 1.2.5 Observing Processes Over Time

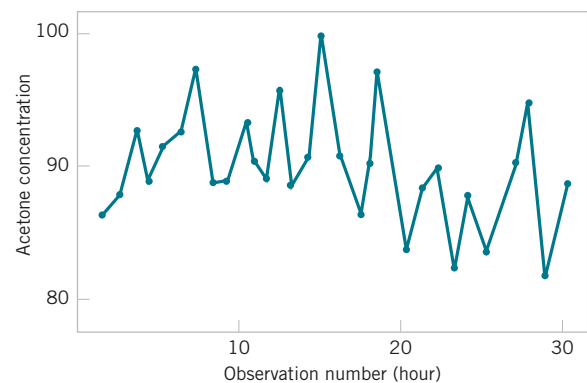
Often data are collected over time. In this case, it is usually very helpful to plot the data versus time in a **time series plot**. Phenomena that might affect the system or process often become more visible in a time-oriented plot and the concept of stability can be better judged.

Figure 1.8 is a dot diagram of acetone concentration readings taken hourly from the distillation column described in Section 1.2.2. The large variation displayed on the dot diagram indicates considerable variability in the concentration, but the chart does not help explain the reason for the variation. The time series plot is shown in Figure 1.9. A shift in the process mean level is visible in the plot and an estimate of the time of the shift can be obtained.

**W. Edwards Deming**, a very influential industrial statistician, stressed that it is important to understand the nature of variability in processes and systems over time. He conducted an



**FIGURE 1.8** The dot diagram illustrates variation but does not identify the problem.



**FIGURE 1.9** A time series plot of concentration provides more information than the dot diagram.